THEORETICAL INVESTIGATION OF JET PROPULSION

A THESIS

Submitted in partial fulfillment
of the requirements for the Degree
of Master of Science in Aeronautical Engineering

by

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Atlanta, Georgia
1942

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Approved:

Date Approved by Chairman May 12, 1942

ACKNOWLEDGMENTS

The author wishes to thank Professor Montgomery Knight who suggested the problem and made many helpful criticisms during its development.

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SUMMARY

The analytical investigation presented in this paper was made to determine the efficiency that could be obtained from an air and gasoline burning jet, particularly at reasonably high velocities such as might be obtained by placing the jet at the tip of a helicopter rotor.

The analysis is in three parts: (1) the performance of a free jet, (2) the augmentation necessary to obtain various overall efficiencies, and (3) a possible method of obtaining the necessary augmentation.

It was found that a free gasoline and air burning jet is inherently inefficient because of the extremely high exhaust velocity which causes a considerable loss of kinetic energy. It is necessary to employ some method of augmenting the thrust of the jet in order to obtain acceptable efficiencies. Calculations of the thrust obtained from a high speed induced flow wind tunnel indicated a possible method of obtaining the necessary augmentation.

INTRODUCTION

The term "jet propulsion", as commonly understood, implies the use of a small, intense jet maintained by some means other than an airscrew. Jet propellers in this restricted sense have a low propulsive efficiency as compared to the air airscrew. Owing to the high discharge velocity of the jet, a comparatively small momentum, or thrust, is obtained for a given amount of kinetic energy in the jet. If, however, some of this kinetic energy could be transmitted to the surrounding air in such a way as to reduce the velocity and increase the momentum of the jet, the thrust and consequently the efficiency would be improved.

To determine what may be done by way of augmentation, or increasing the thrust, it will be helpful to examine a free jet flowing from a nozzle.

The most apparent motion of the jet is an axial one which is initially imparted to the fluid by the pressure in the nozzle. A closer examination will show that there exists rotary motion of the eddies which make up the turbulent boundary between the jet and the surrounding air. Owing to the turbulence, there is a certain amount of mixing between the jet and its surroundings, causing the acceleration of the fluid adjacent to the jet. Thus an inflow of air normal to the jet is set up to replace that carried downstream by the jet.

The thrust might be augmented by directing axially this inflow of air. The Melot augmentor is of this type. It consists of a series of annular guides of curved profile surrounding the jet, the last and largest of which has a diverging cone attached to it making it the shape of a Venturi tube. In testing a Melot-type augmentor, Jacobs and Shoemaker[1] obtained an augmentation of approximately 1.38.

In this paper the performance of a free gasoline and air burning jet was analyzed much the same as in National Advisory Committee for Aeronautics Technical Report No. 159.[2]

After computing the efficiency of the free jet, values of the augmentation necessary to obtain various overall efficiencies were calculated. These calculations were made neglecting the efficiency of the compressor which would be required to supply the combustion chamber air.

It was suggested that an annular nozzle, the exhaust velocity from which would induce a flow inside the annulus, might produce the necessary augmentation. The induced air and the gases from the nozzle would mix and then exhaust through a diffuser. This was the method used to induce a flow in the high speed induced wind tunnel tested by Bailey and Wood.[3] An attempt was made to calculate the augmentation produced by this tunnel and the results were compared with those obtained experimentally by J. J. Harper[4] from an augmentor of this type.

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[1] Eastman N. Jacobs and James M. Shoemaker, Tests on Thrust Augmentors for Jet Propulsion, U.S. National Advisory Committee for Aeronautics, Technical Note No. 431, 1932.

[2] Edgar Buckingham, "Jet Propulsion for Airplanes", U.S. National Advisory Committee for Aeronautics, Annual Report (Technical Report No. 159), 1923.

[3] A. Bailey and S.A. Wood, "The Development of a High Speed Induced Wind Tunnel", Aeronautical Research Committee Reports and Memoranda No.1791

[4] J.J. Harper, Jet Propulsion Experiments. Thesis submitted for Master's Degree in Aero. Engr., Georgia School of Technology, 1942.

RESULTS AND DISCUSSION

Performance of a Free Jet

The performance of a free jet exhausting from a gasoline and air burning chamber will be discussed in eight steps:

1. The temperature of the air delivered by the compressor.
2. The combustion chamber temperature.
3. The exhaust temperature.
4. Jet velocity.
5. Static thrust.
6. Thermal efficiency.
7. Propulsive efficiency.
8. Overall efficiency.

1. Temperature of the air delivered by the compressor. It is assumed that the air in the process of compression from atmospheric pressure to combustion chamber pressure follows the adiabatic law

$$p v^k = \text{constant} \tag{1}$$

where p = absolute pressure

v = specific volume

$k = C_p / C_v$

C_p = specific heat at constant pressure

C_v = specific heat at constant volume

The variations of C_p and C_v with temperature are taken into account for each process, although during the compression the temperature range is such that the variations are almost negligible. Expressing C_p and C_v as functions of the absolute temperature [5]

$$C_p = M' + BT + CT^2 \qquad (2)$$

$$C_v = M + BT + CT^2$$

and for air $M' = 0.239$

$$M = 0.1705$$

$$B = 0$$

$$C = 4.138 \times 10^{-9}$$

now using the familiar gas equation

$$pv = RT \qquad (3)$$

the relation between the temperatures and pressures at any two points during an adiabatic compression becomes

$$P_1/P_0 = \left(T_1/T_0\right)^{\frac{k}{k-1}}$$

or

$$P_1/P_0 = \left(\frac{t_1 + 460}{t_0 + 460}\right)^{\frac{k}{k-1}}$$

since T_1 and T_0 are in degrees F. absolute and t_1 and t_0 are in degrees F. The subscript "0" represents atmospheric conditions, the subscript "1" represents the condition of the air after compression, and the subscript "2" represents combustion chamber conditions.

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[5] Barnard, Ellenwood, and Hirshfield, Heat-Power Engineering, p.81.

$$p_1 = p_2$$

since the air and gasoline enter the chamber under constant pressure.

Then

$$p_2/p_0 = \left(\frac{t_1 + 460}{t_0 + 460}\right)^{\frac{k}{k-1}}$$

this may be written

$$p_2/p_0 = \left(\frac{t_1 + 460}{t_0 + 460}\right)^{\frac{c_p}{c_p - c_v}} \qquad (4)$$

$$c_p - c_v = M' - M$$

= .0685 from equation (2), and using the average value of c_p over the range t_0 to t_1 equation (4) becomes

$$p_2/p_0 = \left(\frac{t_1 + 460}{t_0 + 460}\right)^{\frac{\frac{1}{2}(c_{p_0} + c_{p_1})}{.0685}}$$

Taking the average value of the specific heat as $\frac{1}{2}(c_{p_0} + c_{p_1})$ is not strictly correct since c_p varies as the square of the temperature; however, the error is small enough to be disregarded. Substituting $t_0 = 59\ °F$ the relation between the pressures and temperatures is finally

$$p_2/p_0 = \left(\frac{t_1 + 460}{519}\right)^{3.49 + 15.1 \times 10^{-9}(t_1 + 979)^2} \qquad (5)$$

where p_2/p_0 = combustion chamber pressure in atmospheres

t_1 = temperature of air delivered by compressor in $°F$

The values of t_1 and P_2/P_0 are given in Table I, and Figure 1 shows the variation of t_1 for values of P_2/P_0 from 1 to 8 atmospheres. The range of t_1 from 59°F at $P_2/P_0 = 1$ to 478°F at $P_2/P_0 = 8$ is small enough so that c_p could have been assumed constant.

2. <u>The combustion chamber temperature.</u> It is assumed that air and gasoline are at the temperature t_1 before entering the combustion chamber. A further assumption must be made as to the amount of heat lost in cooling and radiation. In ordinary gasoline motors about one-fourth to one-third of the heat developed in the cylinders is lost to jacket water. In the combustion chamber here contemplated, the temperature will be much higher than the mean in a motor cylinder, and both the chamber and the nozzle will require artificial cooling. However a refractory lining may be used and the chamber allowed to run very hot, so that the unavoidable heat loss will probably be a much smaller fraction than in the usual motor cylinder. Assuming that one-tenth is a sufficient allowance, the remaining fraction which is effective in heating the gas mixture will be $\epsilon = .90$, where ϵ is designated as the "receiver efficiency".

The temperature of the combustion chamber will be

$$t_2 = t_1 + \frac{\epsilon h}{(m+1)\, c_p} \qquad (6)$$

where h = heating value of the gasoline

m = mixture ratio

In calculating the chamber temperature, the lower heating value of the gasoline is used as the water formed on combustion will become superheated steam.

The mixture ratio used is $m = 15$, which is about the value for gasoline motors, and the heating value used is $h = 18675 \frac{B.T.U.}{lb.}$. In this case the temperature range is large enough to necessitate taking into account the variation of the specific heat and equation (5) becomes

$$t_2 = t_1 + \frac{1050}{16\left(\frac{c_{p_1} + c_{p_2}}{2}\right)} \tag{7}$$

Writing equation (7) in terms of the temperatures only by putting

$$C_p = M' + BT + CT^2$$

$$t_2^3 + (t_1 + 1840)\,t_2^2 + (2.138 \times 10^8 - t_1^2)\,t_2 + \left[t_1(t_1^2 + 1840\,t_1 - 2.302 \times 10^8) - 10150 \times 10^8\right] = 0 \tag{8}$$

Equation (8) is a cubic in t_2 and must be solved by successive approximations. For values of P_2/P_0 the corresponding values of t_1 were read from Figure 1 and the values of t_2 calculated from equation (8).

The values of t_2 for various values of P_2/P_0 are given in Table II, and these values are shown graphically by Figure 2. t_2 ranges from 4023° F at $P_2/P_0 = 1$ to 4330° F at $P_2/P_0 = 8$ atmospheres. The variation in t_2 with chamber pressure is due only to the increase in t_1 with chamber pressure. The curve of t_2 could have been plotted very closely by merely adding the values of t_1 at various chamber pressures to the value of $(t_2 - t_1)$ at $P_2/P_0 = 1$.

As anticipated, the values of t_2 are much higher than the mean in a motor cylinder, and some sort of refractory lining for the combustion chamber and nozzle would obviously be required.

3. <u>The exhaust temperature.</u> The expansion of the gases after combustion would be reversibly adiabatic if there were no heat lost to the nozzle walls and if there were no nozzle resistance. The heat loss in the nozzle will probably be negligible but the resistance will not. The expansion will therefore be irreversible and take place according to the equations [6]

$$pv^n = constant \tag{9}$$

where

$$n = \frac{k}{z^2 + k(1-z^2)} \tag{10}$$

z is the coefficient of velocity and is equal to the square root of the nozzle efficiency

$$z = \sqrt{e_n}$$

A nozzle efficiency $e_n = .92$ is assumed over the entire range of chamber pressures. The subscript "3" is used to represent the condition of the expanded gases leaving the jet. With the jet discharging to atmospheric pressure

$$P_3 = P_0$$

From equations (3) and (9)

$$t_3 = \frac{t_2 + 460}{(P_2/P_0)^{\frac{n-1}{n}}} - 460 \tag{11}$$

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6 Buckingham, <u>Op. cit.</u> p.79

Equation (10) may be written

$$n = \frac{c_p}{c_p - e_n(c_p - c_v)}$$

$c_p - c_v$ = constant = .0685 as before, and putting $e_n = .92$

$$n = \frac{c_p}{c_p - .063}$$

Therefore $\frac{n-1}{n} = \frac{.063}{c_p}$

substituting this value in equation (11)

$$t_3 = \frac{t_2 + 460}{\left(P_2/P_0\right)^{\frac{.063}{c_p}}} - 460$$

expressing c_p in terms of the average value from t_2 to t_3

$$t_3 = \frac{t_2 + 460}{\left(P_2/P_0\right)^{\frac{.126}{c_{p_2} + c_{p_3}}}} - 460$$

then writing c_{p_2} and c_{p_3} in terms of the temperatures t_2 and t_3

$$t_3 = \frac{t_2 + 460}{\left(P_2/P_0\right)^{\frac{1}{3.195 + 16.42 \times 10^{-9}\left[t_3 + (t_2 + 920)\right]^2}}} - 460 \quad (12)$$

t_3 may be solved for from equation (12) using the method of successive approximations. Values of t_3 for corresponding values of P_2/P_0 are listed in Table III. Figure 3 is a curve of t_3 vs. P_2/P_0. t_3 varies from 4023° F. at $P_2/P_0 = 1$ to 2655° F. at $P_2/P_0 = 8$.

Even at the higher chamber pressures the exhaust temperature is high enough to necessitate a refractory lining in at least a portion of the nozzle.

4. Jet velocity. By assuming a negligible velocity of the gases just before entering the nozzle the jet velocity may be calculated by means of the equation [7]

$$V_J = 8.02\sqrt{e_n}\sqrt{\frac{k}{k-1}RT_2\left[1-\left(P_0/P_2\right)^{\frac{k-1}{k}}\right]} \quad (13)$$

or alternatively from

$$V_J = 8.02\sqrt{\frac{k}{k-1}RT_2\left[1-\left(P_0/P_2\right)^{\frac{n-1}{n}}\right]} \quad (14)$$

Equation (9) states

$$Pv^n = constant$$

From this relation and equation (3) the relation between P_0/P_2 and T_3/T_2 may be obtained:

$$\left(P_0/P_2\right)^{\frac{n-1}{n}} = T_3/T_2 \qquad \text{since } P_3 = P_0$$

substituting this expression for $\left(P_0/P_2\right)^{\frac{n-1}{n}}$ in equation (14)

$$V_J = 8.02\sqrt{\frac{k}{k-1}R\left(T_2-T_3\right)}$$

or

$$V_J = 8.02\sqrt{\frac{k}{k-1}R\left(T_2-T_3\right)} \quad (15)$$

putting

$$\frac{k}{k-1} = \frac{c_p}{c_p-c_v} = \frac{c_p}{.0685}$$

$$V_J = 8.02\sqrt{\frac{c_p}{.0685}R\left(t_2-t_3\right)} \quad (16)$$

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[7] Kiefer and Stuart, Principles of Engineering Thermodynamics New York: John Wiley and Sons, Inc., 1930. pp.243-244.

again expressing C_p in terms of the temperature t_2 and t_3 ; and putting $R = 53.3$, the value of the gas constant for air

$$V_J = 58.6\sqrt{\left[3.49+15.1\times10^{-9}(t_2+t_3+920)^2\right](t_2-t_3)} \qquad (17)$$

The jet velocity is readily calculated from equation (17) as the values of t_2 and t_3 for various chamber pressures have already been obtained.

Table IV gives the values of V_J and P_2/P_0 used in plotting Figure 4. The jet velocity reaches a value of 5060 Ft./sec. at $P_2/P_0 = 8$ atmosphere.

5. <u>Static thrust</u>. The static thrust, T_S , will be equal to the momentum per second acquired by the gases in expanding from combustion chamber pressure to atmospheric pressure.

$$T_S = M V_J \qquad (18)$$

M = mass rate of flow of fuel and air

V_J = jet velocity

Caloulating T_S on the basis of a fuel consumption of $1\ ^{lb}/_{hr}$

$$M = \frac{1}{g}\left(\frac{m+1}{3600}\right)$$

$$M = .000138 \ \frac{slugs}{sec.}$$

Therefore the static thrust in pounds is

$$T_S = .000138\ V_J \qquad (19)$$

where V_J is in feet per second.

It is thus only necessary to multiply the values of the jet velocity by a constant in order to obtain the static thrust.

Figure 5 shows the variation in static thrust with chamber pressures and the values are listed in Table V. At $P_2/P_0 = 8$ atmospheres a static thrust of .698 lb. is obtained for a fuel consumption of $1 \frac{lb.}{hr.}$.

6. <u>Thermal efficiency.</u> The thermal efficiency will be

$$\eta_{TH} = \frac{T_s V_J}{778\, H} \tag{20}$$

where H is the heat supplied to the combustion chamber in $\frac{B.T.U.}{sec.}$.
To be exact in computing the efficiency the higher heating value of the gasoline should be used, and the heat acquired by the air during compression should be included. Therefore

$$H = \frac{h'}{3600} + \frac{m \Delta h_{air}}{3600} \tag{21}$$

for a full consumption of $1 \frac{lb.}{hr.}$.

$$\Delta h_{air} = c_p (t_1 - t_0)$$

t_0 was chosen as 59° F and it was found that c_p could be considered constant over the range of t_0 to t_1 for the pressure used, therefore putting $c_p = .241$

$$\Delta h_{air} = .241\,(t_1 - 59)$$

Taking the higher heating value of gasoline as $20{,}000 \ \frac{B.T.U.}{lb.}$ equation (21) becomes

$$H = \frac{20000}{3600} + \frac{15(.241)(t_1-59)}{3600}$$

$$H = 5.55 + .001005(t_1 - 59) \qquad (22)$$

Substituting this value of H in equation (20)

$$\eta_{TH} = \frac{T_S V_J}{4325 + .78(t_1 - 59)} \qquad (23)$$

7. <u>Propulsive efficiency</u>. In calculating the propulsive efficiency the static thrust was taken as the thrust at each velocity of the body to which the chamber is attached. This would be the case if the jet were discharging at the tip of a helicopter rotor blade and the air for compression were being taken in at some point on the fuselage. The propulsive efficiency would then be

$$\eta_{PROP} = \frac{T_S V}{T_S V_J}$$

or

$$\eta_{PROP} = \frac{V}{V_J} \qquad (\qquad (24)$$

where $V =$ tip velocity of the rotor.

8. <u>Overall efficiency</u>. The overall efficiency, neglecting the compressor, would be

$$\eta = \eta_{TH} \times \eta_{PROP}$$

$$\eta = \frac{T_S V}{778\, H} \qquad (25)$$

Values of η_{TH} , η_{PROP} and η for values of V from 100 to 1300 $ft/_{sec.}$ are given in Table VI. Figure 6 shows the variation of η with chamber pressure for the values of V in Table VI. The efficiency increases quite rapidly from $P_2/P_0 = 1$ to about 2.5 atmospheres, but much more slowly from that point to P_2/P_0 = 8 atmospheres. At the lower values of V the efficiencies are of course very low.

Augmentation Necessary to Obtain Various Overall Efficiencies

The augmentation necessary to obtain a given overall efficiency will, of course, vary with the velocity of the body, V . In calculating the augmentation necessary to obtain the various efficiencies two values of V were chosen, 500 and 900 $ft/_{sec.}$.

The augmentation necessary to obtain a certain efficiency is merely a matter of dividing the desired efficiency by the overall efficiency at the chamber pressure and velocity, V , under consideration. This gives the amount the thrust must be augmented as may be seen from equation (3),

$$\eta = \frac{T_s V}{778 H}$$

Since

$$\eta_{AVG.} = \frac{T_{AVG.} V}{778 H}$$

augmentation

$$\eta_{AVG.}/\eta = T_{AVG.}/T_s$$

The values of augmentation necessary to obtain overall efficiencies of 5, 10, 15, 20 and 25% are given in Tables VII and VIII. The curves of augmentation vs. chamber pressure for the several overall efficiencies are shown in Figures 7 and 8.

It was thought that curves of the augmentation necessary to obtain various efficiencies would be of value if plotted against V_J, nozzle velocity of the free jet. Figures 9 and 10 are the augmentation values from Figures 7 and 8 re-plotted against V_J.

It is also interesting to further consider the case of a forward velocity of 900 ft./sec. From the equation[8]

$$P_s/P_o = 1 + \frac{kV_o^2}{2C_o^2}\left[1 + \frac{V_o^2}{4C_o^2} - \frac{(k-2)}{24}\frac{V_o^4}{C_o^4} + \text{---}\right] \quad (26)$$

which gives the ratio of stagnation point pressure to free stream static pressure, a free stream velocity of 900 ft./sec. produces a compression under standard atmospheric conditions, 59° F. and 29.92 in Hg. of 1.53 atmospheres. If the combustion chamber were operated at this pressure and the compression obtained by utilizing the free stream velocity, the thrust of the jet would be lowered by the amount $\frac{mV}{3600g}$ for the assumed fuel consumption of $1\ \frac{lb.}{hr.}$. As before, m = mixture ratio = 15. Now for $V = 900$ ft./sec.

$$\frac{mV}{3600g} = .116 \text{ lb.}$$

From Figure 4 at $P_c/P_o = 1.53$

$$T_s = .343 \text{ lb.}$$

subtracting $\frac{mV}{3600g}$

8 R. A. Dodge and M. J. Thompson, Fluid Mechanics (New York: McGraw-Hill Book Company, Inc., 1937) p.369.

$$T_s' = .343 - .116 = .227$$

From Figure 5 at $P_e/P_o = 1.53$, $V = 900$ ft./sec., $\eta = 7.15\%$

Multiplying this value by T_s'/T_s, the efficiency becomes

$$\eta' = 7.15 \; \frac{.227}{.343} = 4.73\%$$

Therefore utilizing the velocity to produce compression, an augmentation of $\frac{20}{\eta'} = \frac{20}{4.73} = 4.225$ is needed to obtain an overall efficiency of 20%. This value, as well as the augmentation necessary to obtain efficiencies of 10, 15 and 25%, are given in Table IX and plotted as points on Figures 8 and 10.

Possible Method of Obtaining Augmentation

As mentioned in the introduction, an augmentation of 1.4 seems to be the maximum heretofore obtained. This is approximately the value obtained for the Melot-type augmentor tested by Jacobs and Shoemaker.[9]

An alternative of the Melot augmentor is one in which an annular nozzle is used. A flow is induced inside the annulus by the high exhaust velocity of the nozzle. The induced air mixes with the exhaust of the nozzle, and the mixture exhausts through a diffuser. This method of inducing a flow was used in the high speed wind tunnel of Bailey and Wood.[10] Figure 11 is a sketch of this tunnel. The working portion is of rectangular cross-section, 6 inches by 3 inches. The nozzle gap is .04 inches.

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[9] Jacobs and Shoemaker, op. cit.

[10] Bailey and Wood, op. cit.

An attempt was made to calculate the augmentation obtained at various pressures in the high pressure chamber. The only data available for making these calculations were velocity surveys in the tunnel. Table X gives the ratio of induced velocity to the local velocity of sound for various pressures in the high pressure chamber. These values were obtained at a point 2 3/8 in. upstream of the plane of the nozzle, or exactly at the downstream end of the working portion of the tunnel. It was necessary to assume atmosphere conditions. Standard conditions of 59° F. and 29.92 in. Hg. were used.

The calculated values of the augmentation of the tunnel are given in Table XI and plotted in Figures 12 and 13. In Figure 12 the augmentation is plotted against chamber pressure and in Figure 13 against the velocity of a free jet exhausting from a chamber at the same pressures as those of the high pressure chamber of the tunnel.

In calculating the augmentation, the exhaust pressure of the nozzle had to be found. This was taken as the static pressure at the point 2 3/8 in. upstream from the plane of the nozzle, the point at which the induced velocity is given in Table X. The static pressure would undoubtedly vary slightly from this point to the nozzle but a small variation in exhaust pressure would have very little effect on the exhaust velocity of the nozzle. The symbols used are as follows: atmospheric conditions

P_0 = absolute pressure

ρ_0 = density

T_0 = absolute temperature

for the induced air just upstream of the nozzle

V_1 = velocity

c_1 = local velocity of sound

p_1 = absolute pressure

ρ_1 = density

T_1 = absolute temperature

M_1 = mass rate of flow

condition of the air in high pressure chamber

p_2 = absolute pressure

T_2 = absolute temperature

the exhaust from high pressure chamber

V_J' = jet velocity

p_J' = absolute pressure

ρ_J' = density

T_J' = absolute temperature

M_J' = mass rate of flow

for the mixture just downstream from the nozzle

V_m = velocity

p_m = absolute pressure

ρ_m = density

T_m = absolute temperature

as the mixture exhausts from the diffuser to atmospheric pressure

V_{m_o} = velocity

p_o = absolute pressure

V_J is velocity of a free jet exhausting to atmospheric pressure.

The equation[11]

$$P_0/P_1 = \left(1 + \frac{k-1}{2}\,\frac{V_1^2 - V_0^2}{C_1^2}\right)^{\frac{k-1}{k}} \tag{27}$$

gives the ratio of the pressures at two points on the same stream line. In this case the velocity, V_0, of the induced flow before entering the tunnel is zero and the equation becomes

$$P_1 = \frac{P_0}{\left(1 + \frac{k-1}{2}\,\frac{V_1^2}{C_1^2}\right)^{\frac{k-1}{k}}} \tag{28}$$

Equation (27) is valid so long as V_1 is less than C_1. For the case where V_1 is greater than C_1, P_0/P_1 may be calculated from Lord Rayleigh's formula[12]

$$\left(P_0/P_1\right)^{\frac{k-1}{k}} = \left(\frac{k+1}{4k}\right)\left(P_c/P_1\right)^{\frac{k-1}{k}}\left[1 + \frac{k-1}{k+1}\,P_1/P_c\right] \tag{29}$$

where

$$P_c/P_1 = \frac{2k}{k+1}\,\frac{V_1^2}{C_1^2} - \frac{k-1}{k+1} \tag{30}$$

and P_c = pressure at the point where the velocity of the air becomes equal to the local velocity of sound. Substituting the values of V_1/C_1 from Table X in equation (28), P_1 can be calculated for various pressures in the high pressure chamber. Then since P_1 and P_J' are considered equal, the exhaust pressure of the nozzle has been found.

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[11] Dodge and Thompson, op. cit.

[12] Lord Rayleigh, Proceedings of the Royal Society, (A), Vol.84 1910, p.283.

The temperature of the high pressure chamber was calculated from the adiabatic relationship

$$T_2 = T_o \left(P_2 / P_o \right)^{\frac{k-1}{k}} \tag{31}$$

Next the jet velocity was found from

$$V_J' = 109.7 \sqrt{T_2 \left[1 - \left(P_1 / P_2 \right)^{\frac{k-1}{k}} \right]} \tag{32}$$

and the temperature of the jet was calculated from

$$T_J' = T_2 \left(P_1 / P_2 \right)^{\frac{k-1}{k}} \tag{33}$$

To find the density of the air exhausting from the nozzle, the gas law $pv = RT$ can be written

$$\rho_J' = \frac{P_1}{gRT_J'} \tag{34}$$

The mass rate of flow of air from the high pressure chamber is then

$$M_J' = V_J' \rho_J' A_J \tag{35}$$

where A_J area of the nozzle and an appromination of .174 sq.in. was made.

The temperature and density of the induced air will be equal to the temperature and density of the air from the nozzle, and thus the temperature and density after mixing will remain unchanged.

To find the velocity of the induced air, the local velocity of sound was calculated from

$$c_1 = \sqrt{\frac{k\,p_1}{\rho_1}} \tag{36}$$

The mass rate of flow of the induced air is then

$$M_1 = V_1\,\rho_1\,A_1 \tag{37}$$

where A_1 = cross-sectional area of the tunnel

= 6 x 3 = 18 sq.in.

The velocity of the mixture was obtained by applying the law of conservation of momentum

$$V_m = \frac{M_1 V_1 + M_J' V_J'}{M_1 + M_J'} \tag{38}$$

$M_1 + M_J'$ is of course the total mass rate of flow, or the mass rate of flow of the mixture, M_m .

The exhaust velocity of the mixture will be

$$V_{m_o} = \sqrt{12030\,T_1\left[1 - \left(\frac{p_o}{p_1}\right)^{\frac{k-1}{k}}\right] + V_m^2} \tag{39}$$

and the thrust of the tunnel is then

$$T_{s_a} = V_{m_o}\,M_m \tag{40}$$

To calculate the augmentation obtained from the tunnel, the thrust of the tunnel was compared with that of a free jet exhausting to atmospheric pressure. The mass rate of flow and chamber pressure were taken equal to those of the high pressure chamber of the tunnel. The velocity of the free jet is

$$V_J = 109.7\sqrt{T_2\left[1-\left(P_0/P_2\right)^{\frac{k-1}{k}}\right]} \qquad (41)$$

and the thrust is then

$$T_s = V_J M_J'$$

Therefore the augmentation is

$$T_{s_a}/T_s$$

From Figure 12, it is indicated that it might be possible to obtain augmentation values of about 3. Figures 12 and 13 also show the experimental results obtained by J. J. Harper.[13] These results were obtained from a set-up quite similar to the high speed tunnel. The maximum value of augmentation obtained was about 1.4, approximately the same as has heretofore been obtained with the Melot type augmentor. Figures 12 and 13 seem inconsistant in that the experimental and calculated curves do not overlap as far when plotted against the velocity of a free jet as when plotted against the chamber pressure. This is due to the fact that in the experiments made by Mr. Harper the temperature of the high pressure chamber was the same as room temperature.

The fact that the experimental results are much lower than the calculated results indicate that an erroneous assumption must have been

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[13] Harper, op. cit.

made. It is possible that the amount of turbulence after mixing would reduce the velocity of the mixture considerably. However, it is also possible that higher augmentation values might be obtained by a more extensive series of tests.

A comparison of Figures 8 and 12 shows that the calculated augmentation values indicate that it might be possible to obtain an overall efficiency of 15% for the case of a forward velocity of 900 ft./sec., utilizing the velocity to produce compression. The experimental results show that it would be possible to obtain an efficiency of approximately only 5% , still using the velocity to produce compression. The comparison of Figures 8 and 12 cannot be strictly made since it is doubtful as to whether the augmentation values would be maintained if the induced air had an initial velocity.

By comparing Figures 10 and 13, it is noted that both the calculated and experimental values of augmentation are obtained at lower jet velocities than the exhaust velocity of an air and gasoline burning chamber. It is possible that the experimental values of augmentation might be increased at higher jet velocities.

The calculated augmentation values fall off at the higher jet velocities. This is due to a decrease in the induced flow, possibly caused by a vena contracta at the plane of the nozzle which is formed as large amounts of air are forced from the nozzle at high chamber pressures. The induced flow could very probably be maintained at the higher chamber pressures by the use of a convergent-divergent nozzle. A convergent nozzle, only, was used in the tunnel. An increase in the ratio of tunnel area to nozzle area might further help to maintain the induced flow at high chamber pressures.

Figure 8 shows that at a forward velocity of 900 feet per second an augmentation of 3 would produce an overall efficiency of 25 %, neglecting compressor efficiency, at a chamber pressure of 1.85 atmospheres, while an augmentation of 1.4 would produce an overall efficiency of 15 % at a chamber pressure of 3.1 atmospheres.

CONCLUSIONS

As the result of this investigation, the following conclusions were arrived at:

(1) A free air and gasoline burning jet is inherently inefficient because of the extremely high exhaust velocity, causing a considerable loss of kinetic energy.

(2) It is necessary to employ some means of augmentation to obtain acceptable efficiencies for the speeds at which airplanes now operate.

(3) Calculations for the thrust obtained from a high speed induced flow wind tunnel indicate a possible method of obtaining the necessary augmentation.

(4) Experimental results fall far below the calculated results but might be increased by more careful design.

(5) Utilizing the forward velocity to produce compression, the calculated values of augmentation indicate an overall efficiency of 15 % might be obtained at a velocity of 900 feet per second.

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TABLE I

P_2/P_0 Atmos.	t_1 °F.
1.306	100
1.748	150
2.315	200
2.996	250
3.802	300
4.776	350
5.888	400
7.195	450
8.71	500

TABLE II

P_2/P_0 Atmos.	t_2 °F.
1	4023
3	4158
5	4234
7	4300

TABLE III

P_2/P_0 Atmos.	t_3 °F.
1	4023
2	3495
3	3230
4	3050
5	2920
6	2810
7	2725
8	2655

TABLE IV

P_2/P_0 Atmos.	V_J Ft./sec.
1	0
1.25	1827
1.5	2440
1.75	2825
2	3100
3	3810
4	4220
5	4500
6	4730
7	4900
8	5060

TABLE V

P_2/P_0 Atmos.	T_s Lb.
1	0
1.25	.252
1.5	.337
1.75	.39
2	.4275
3	.525
4	.5815
5	.6210
6	.6525
7	.676
8	.698

TABLE VI

P_2/P_0	η_{TH}	V = 100 ft./sec.		V = 300 ft./sec.		V = 500 ft./sec.	
		η_{PROP}	η	η_{PROP}	η	η_{PROP}	η
Atmos.	%	%	%	%	%	%	%
1.25	10.58	5.465	.578	16.4	1.735	27.3	2.895
1.5	18.8	-	-	-	-	20.5	3.85
1.75	25.05	-	-	-	-	17.7	4.43
2	30	3.22	.966	9.67	2.90	16.1	4.84
3	44.6	2.62	1.17	7.87	3.51	13.1	5.85
4	54.3	2.37	1.29	7.10	3.87	11.86	6.45
5	61.2	2.22	1.36	6.66	4.08	11.11	6.80
6	67.3	2.11	1.42	6.34	4.26	10.56	7.10
7	71.6	2.04	1.46	6.12	4.38	10.21	7.30
8	76	1.973	1.50	5.91	4.50	9.86	7.50

P_2/P_0	η_{TH}	V = 700 ft./sec.		V = 900 ft./sec.		V = 1100 ft./sec.	
		η_{PROP}	η	η_{PROP}	η	η_{PROP}	η
Atmos.	%	%	%	%	%	%	%
1.25	10.58	38.3	4.05	49.2	5.20	60.1	6.35
1.5	18.8	-	-	36.9	6.93	-	-
1.75	25.05	-	-	31.85	7.98	-	-
2	30	22.55	6.76	29.0	8.70	35.4	10.63
3	44.6	18.35	8.19	23.6	10.55	27.7	12.88
4	54.3	16.6	9.03	21.35	11.62	26.05	14.19
5	61.2	15.65	9.52	20.0	12.25	24.4	14.95
6	67.3	14.8	9.94	19.0	12.80	23.2	15.61
7	71.6	14.3	10.23	18.35	13.15	22.4	16.05
8	76	13.83	10.51	17.77	13.51	21.7	16.5

P_2/P_0	η_{TH}	V = 1300 ft./sec.	
		η_{PROP}	η
Atmos.	%	%	%
1.25	10.58	71	7.51
2	30	41.8	12.56
3	44.6	34	15.33
4	54.3	30.8	16.77
5	61.2	28.8	17.67
6	67.3	27.4	18.45
7	71.6	26.5	18.96
8	76	25.64	19.50

TABLE VII

V = 500 ft./sec.

P_2/P_0 Atmos.	V_J ft./sec.	Augmentation needed to obtain efficiencies of 10 %	15 %	20 %	25 %
1.25	1827	3.45	5.18	6.91	8.63
1.5	2440	2.60	3.895	5.2	6.49
1.75	2825	2.255	3.38	4.51	5.64
2	3100	2.065	3.10	4.13	5.16
3	3810	1.71	2.565	3.42	4.275
4	4220	1.55	2.325	3.10	3.88
5	4500	1.47	2.205	2.94	3.675
6	4730	1.408	2.115	2.82	3.52
7	4900	1.37	2.055	2.74	3.42
8	5060	1.334	2.0	2.665	3.33

TABLE VIII

V = 900 ft./sec.

P_2/P_0 Atmos.	V_J ft./sec.	Augmentation needed to obtain efficiencies of 10 %	15 %	20 %	25 %
1.25	1827	1.922	2.885	3.84	4.81
1.5	2440	1.442	2.164	2.885	3.605
1.75	2825	1.252	1.88	2.505	3.13
2	3100	1.15	1.724	2.30	2.875
3	3810	-	1.421	1.895	2.37
4	4220	-	1.29	1.72	2.15
5	4500	-	1.225	1.631	2.04
6	4730	-	1.172	1.562	1.953
7	4900	-	1.141	1.52	1.901
8	5060	-	1.11	1.480	1.85

TABLE IX

$V = 900$ ft./sec.

P_2/P_0	V_J	Augmentation needed to obtain efficiencies of			
Atmos.	ft./sec.	10%	15%	20%	25%
1.53	2480	2.11	3.17	4.225	5.28

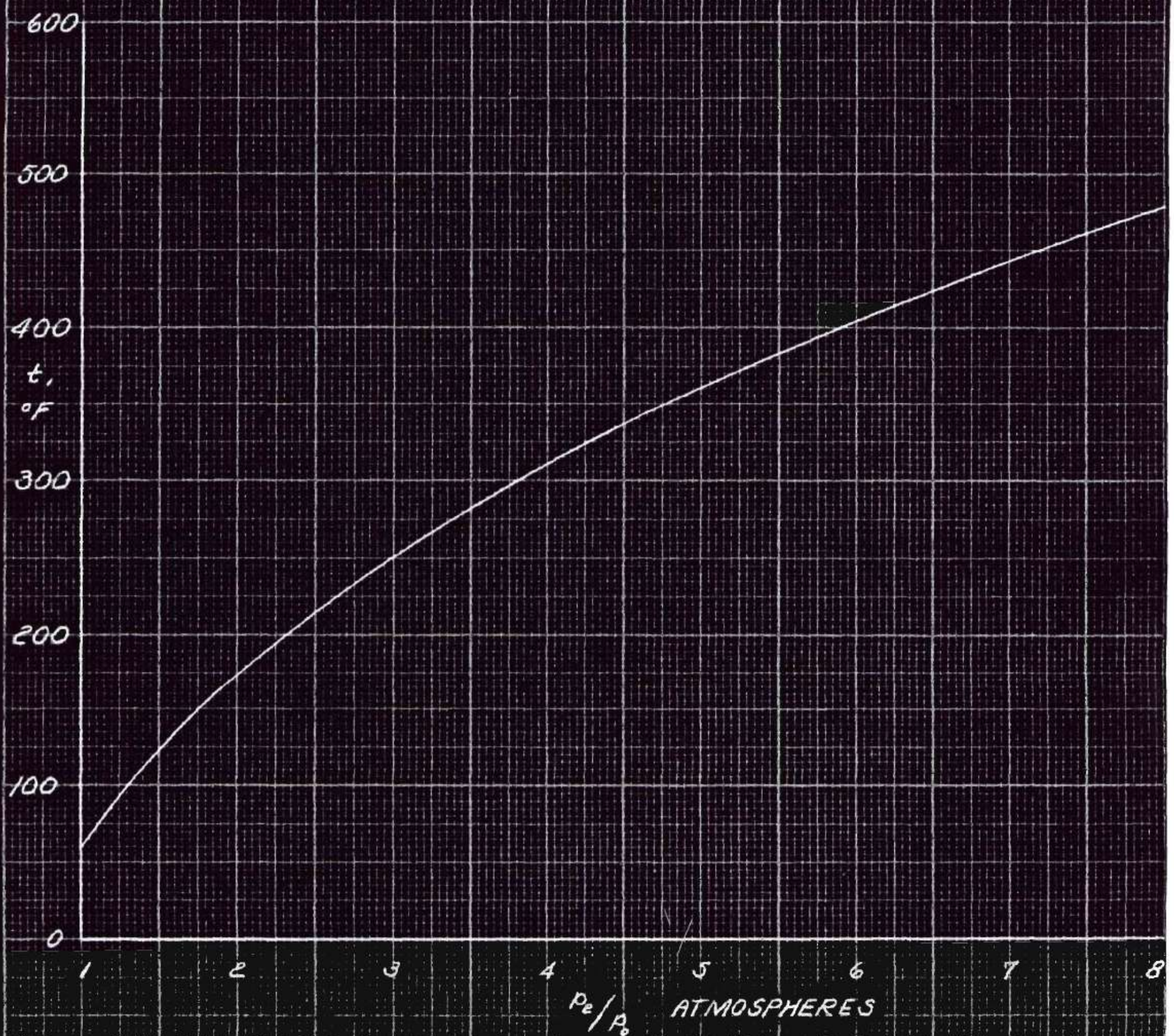
TABLE X

P_2 lb./sq.in. Gage	$\frac{V_1}{C_1}$
20	.76
30	.94
35	.977
40	.957
45	.915
55	.85
65	.80

TABLE XI

P_2/P_0	V_J	Augmentation
Atmos.	ft./sec.	
2.36	1319	3.12
3.04	1530	3.12
3.38	1615	3.075
3.72	1690	2.94
4.065	1758	2.84
4.745	1873	2.64
5.42	1972	2.47

FIG. 1



VARIATION OF TEMPERATURE OF AIR DELIVERED BY COMPRESSOR WITH CHAMBER PRESSURE

$t_o = 59°F$

R.E.W.
11/20/41

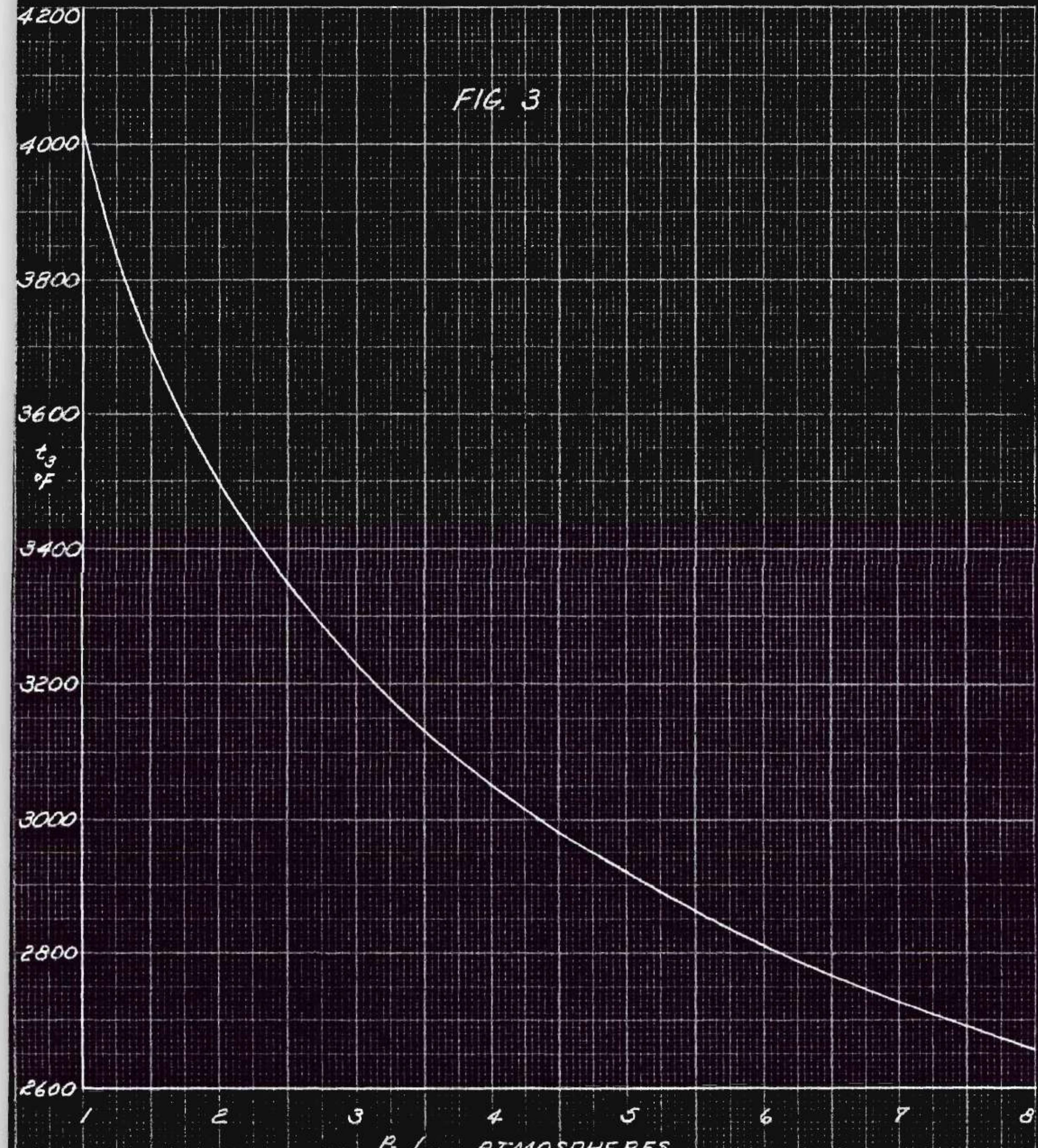
FIG. 2

VARIATION OF COMBUSTION CHAMBER
TEMPERATURE WITH CHAMBER PRESSURE

$t_0 = 59°F$

R.E.W.
11/20/41

FIG. 3

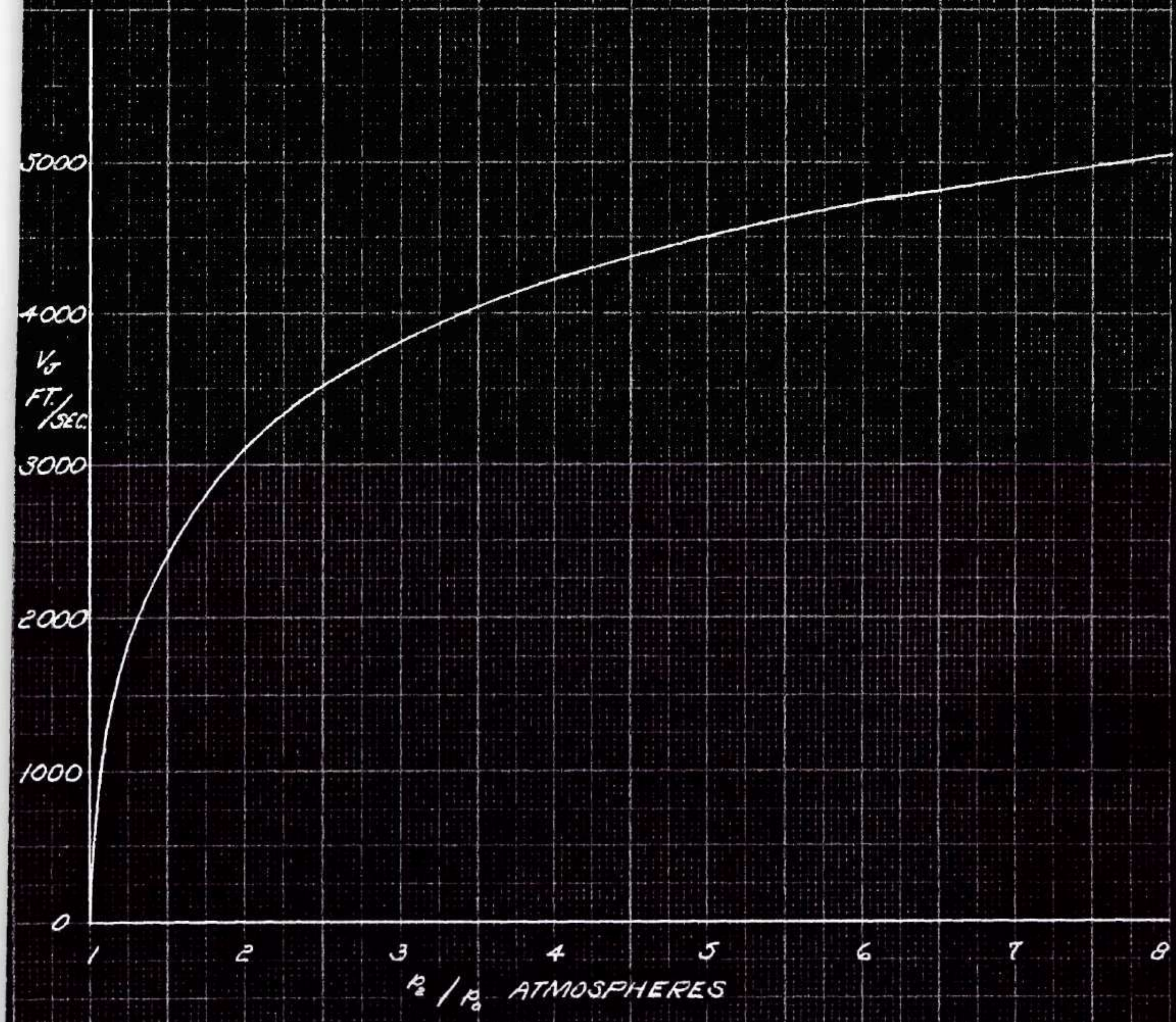


VARIATION OF EXHAUST TEMPERATURE WITH CHAMBER PRESSURE

$t_0 = 59°F$

R.E.W.
11/22/41

FIG. 4

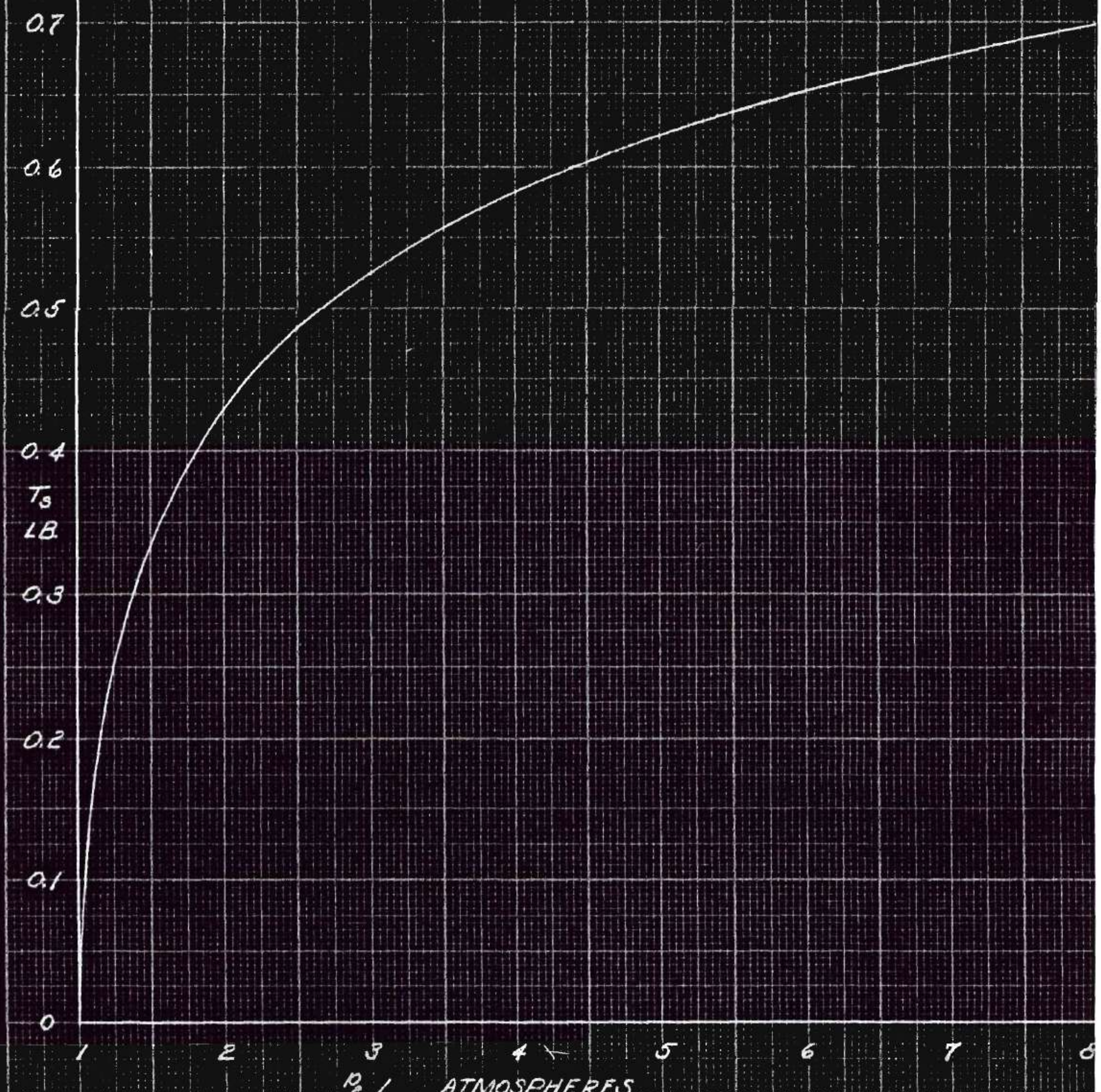


VARIATION OF JET VELOCITY WITH CHAMBER PRESSURE

$t_0 = 59°F$

R.E.W.
11/23/41

FIG. 5



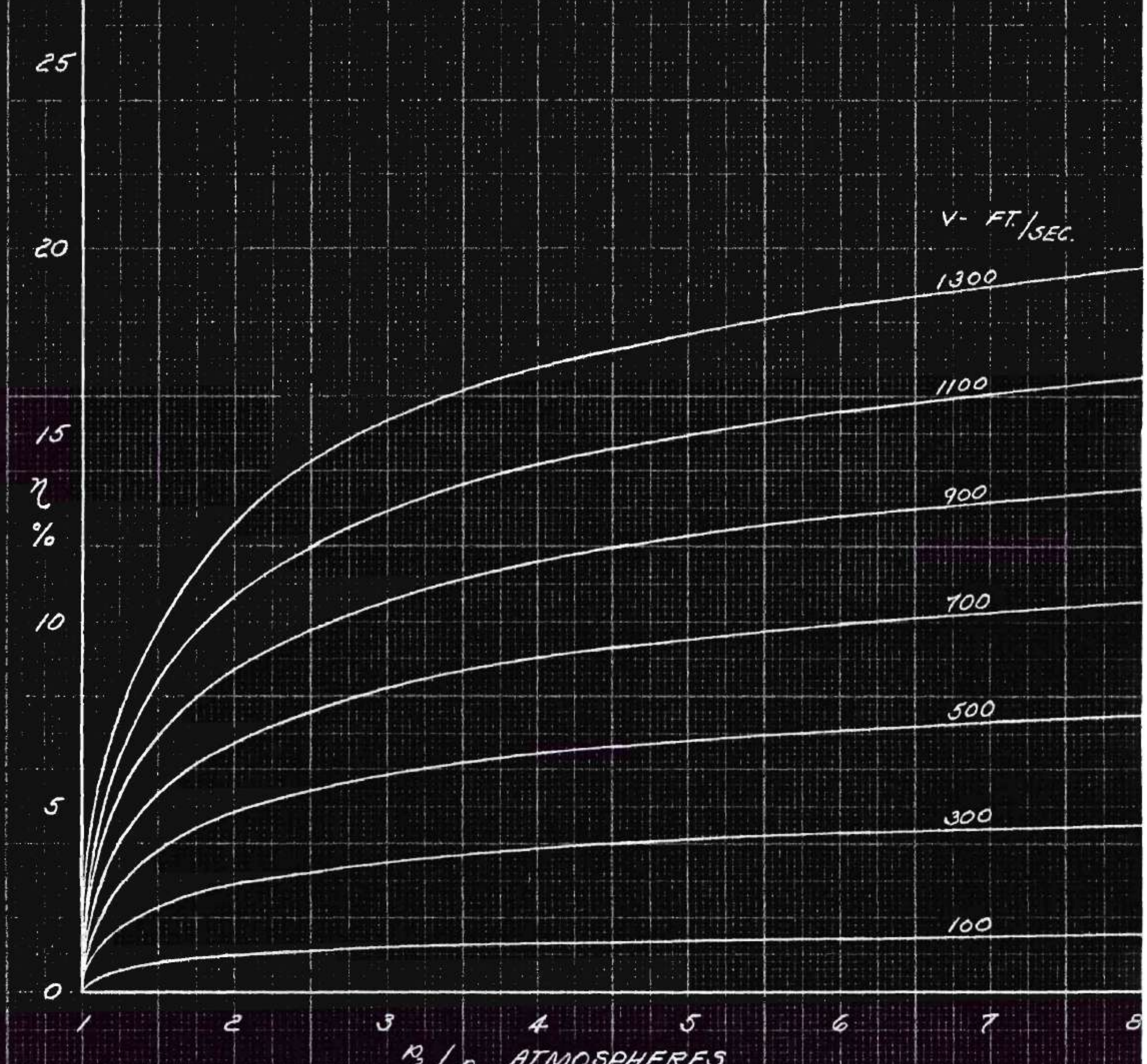
VARIATION OF STATIC THRUST WITH CHAMBER PRESSURE

FUEL CONSUMPTION = 1 LB. PER HR.

$t_o = 59°F$

R.E.W.
11/23/41

FIG. 6

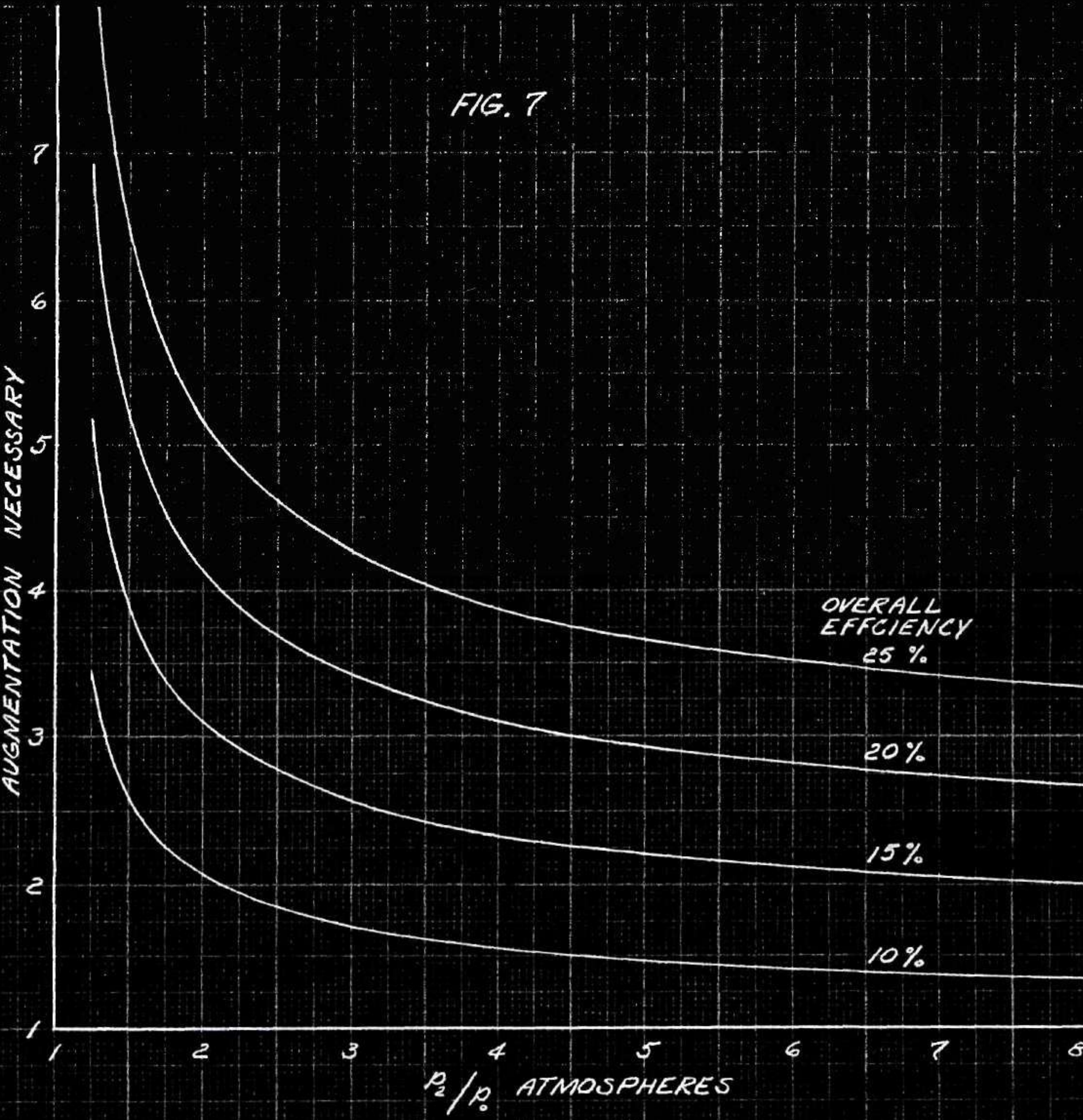


VARIATION OF OVERALL EFFICIENCY WITH CHAMBER PRESSURE - EFFICIENCY OF COMPRESSOR NEGLECTED

$t_0 = 59°F$

R.E.W.
2/24/41

FIG. 7



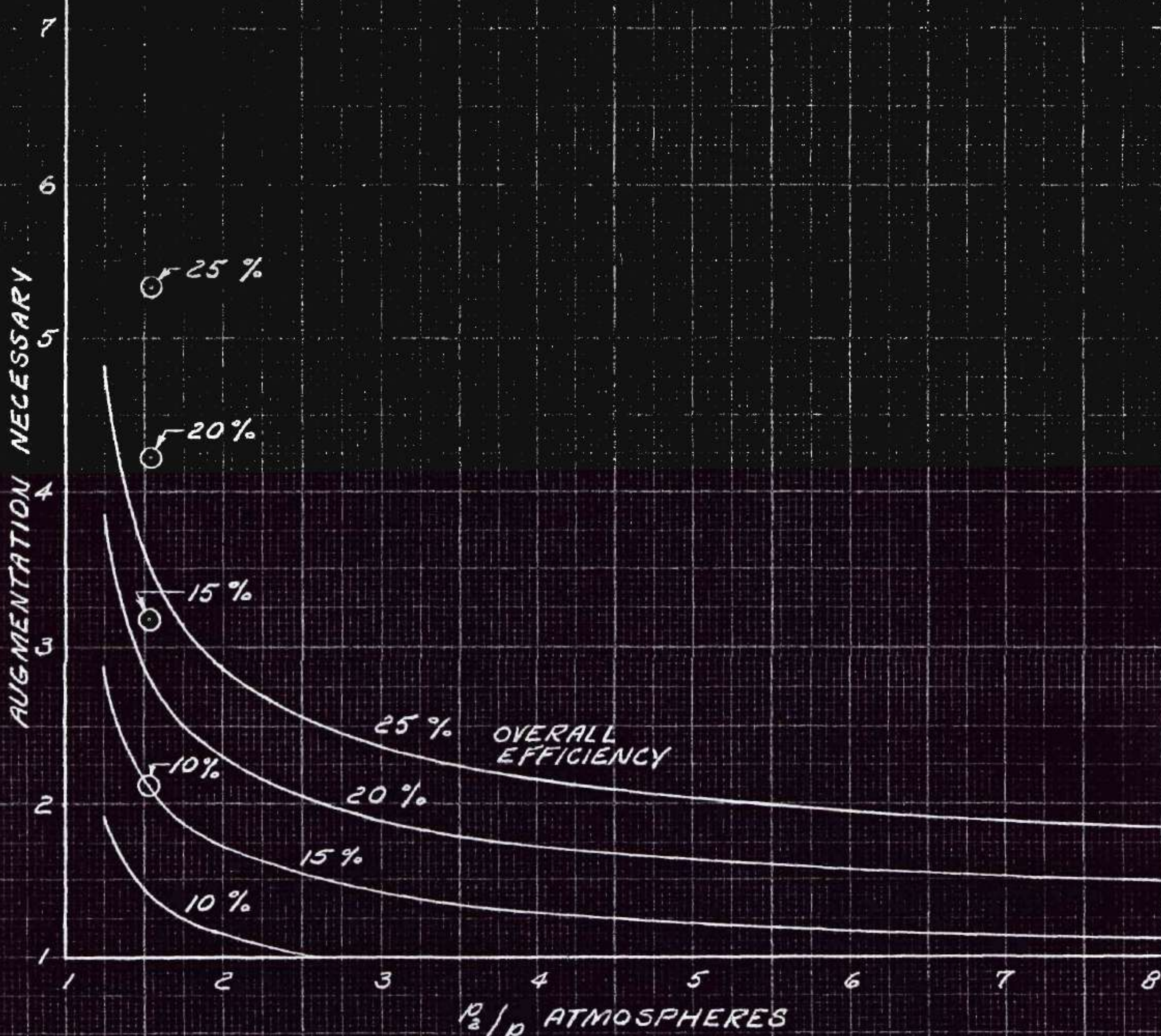
AUGMENTATION NECESSARY TO OBTAIN VARIOUS OVERALL EFFICIENCIES
EFFICIENCY OF COMPRESSOR NEGLECTED

$V = 500$ FT./SEC.

$t_0 = 59°F$

R.E.W.
3/17/42

FIG. 8



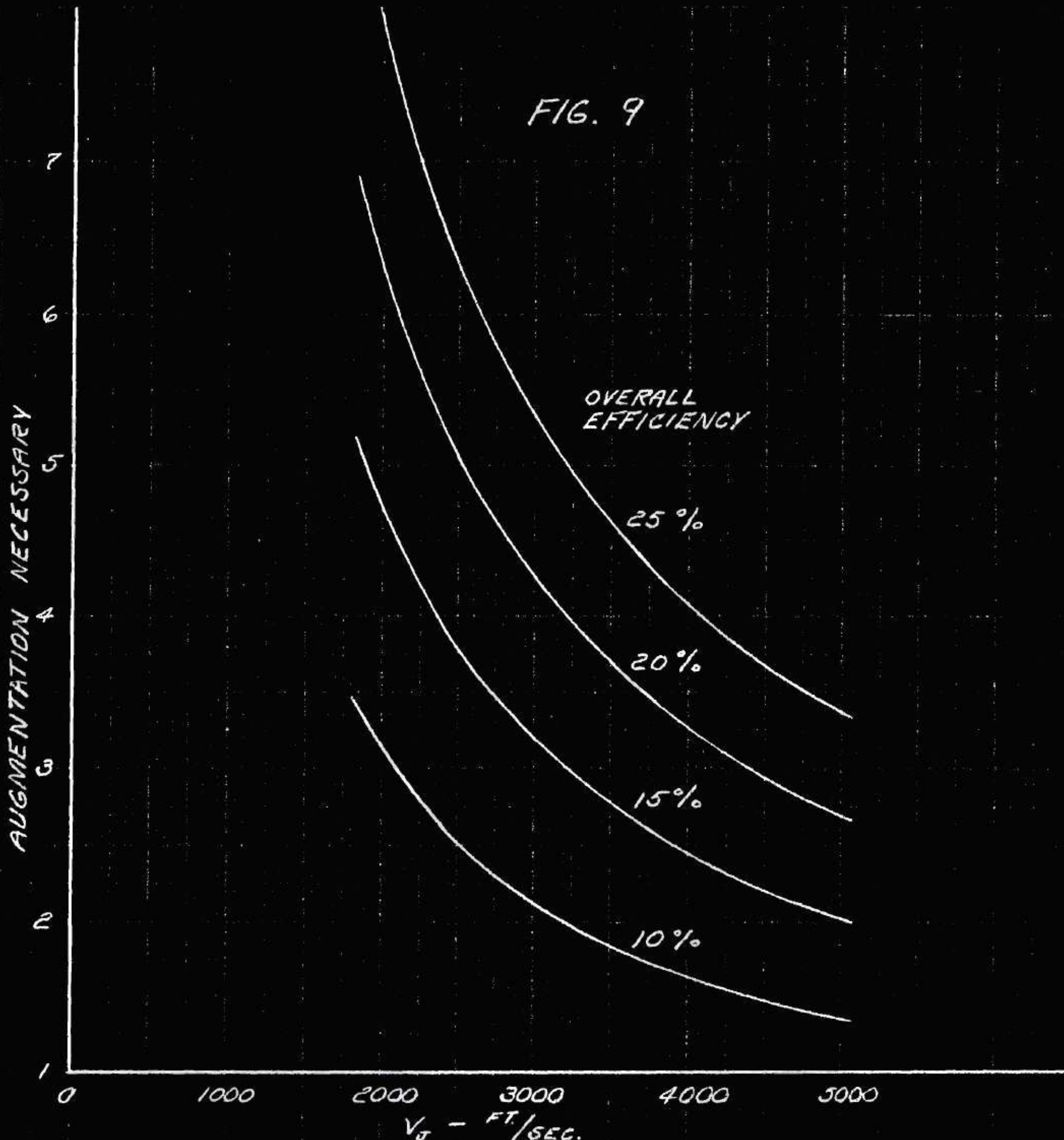
AUGMENTATION NECESSARY TO OBTAIN VARIOUS
OVERALL EFFICIENCIES
EFFICIENCY OF COMPRESSOR NEGLECTED

$V = 900$ FT./SEC.

CIRCLED POINTS — VALUES OBTAINED USING
VELOCITY, V, TO PRODUCE COMPRESSION

$t_0 = 59\,°F$

R. E. W.
4/16/42



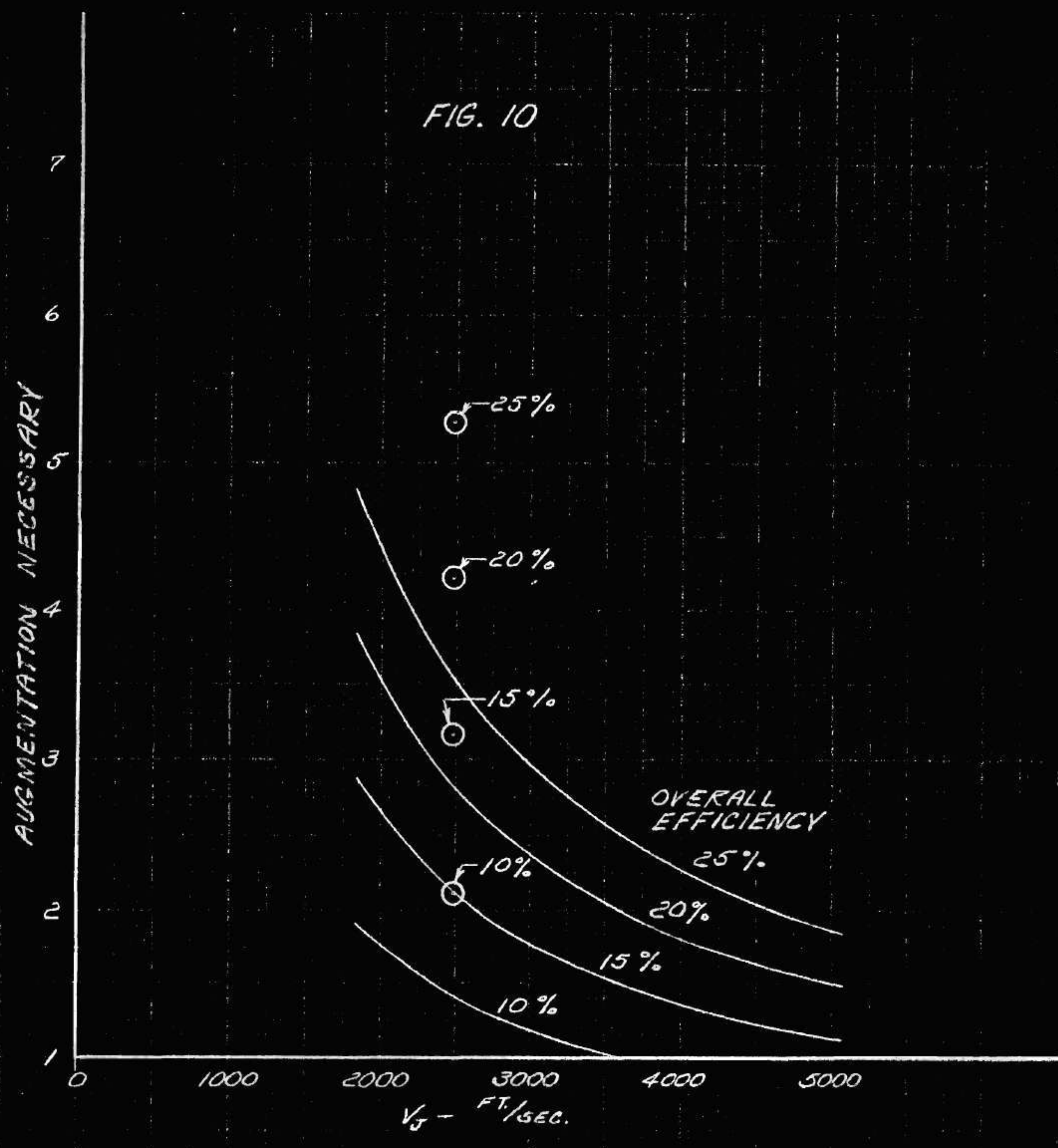
AUGMENTATION NECESSARY TO OBTAIN VARIOUS OVERALL EFFICIENCIES
EFFICIENCY OF COMPRESSOR NEGLECTED

$V = 500$ FT./SEC.

$t = 59°F.$

R.E.W.
4/20/42

FIG. 10



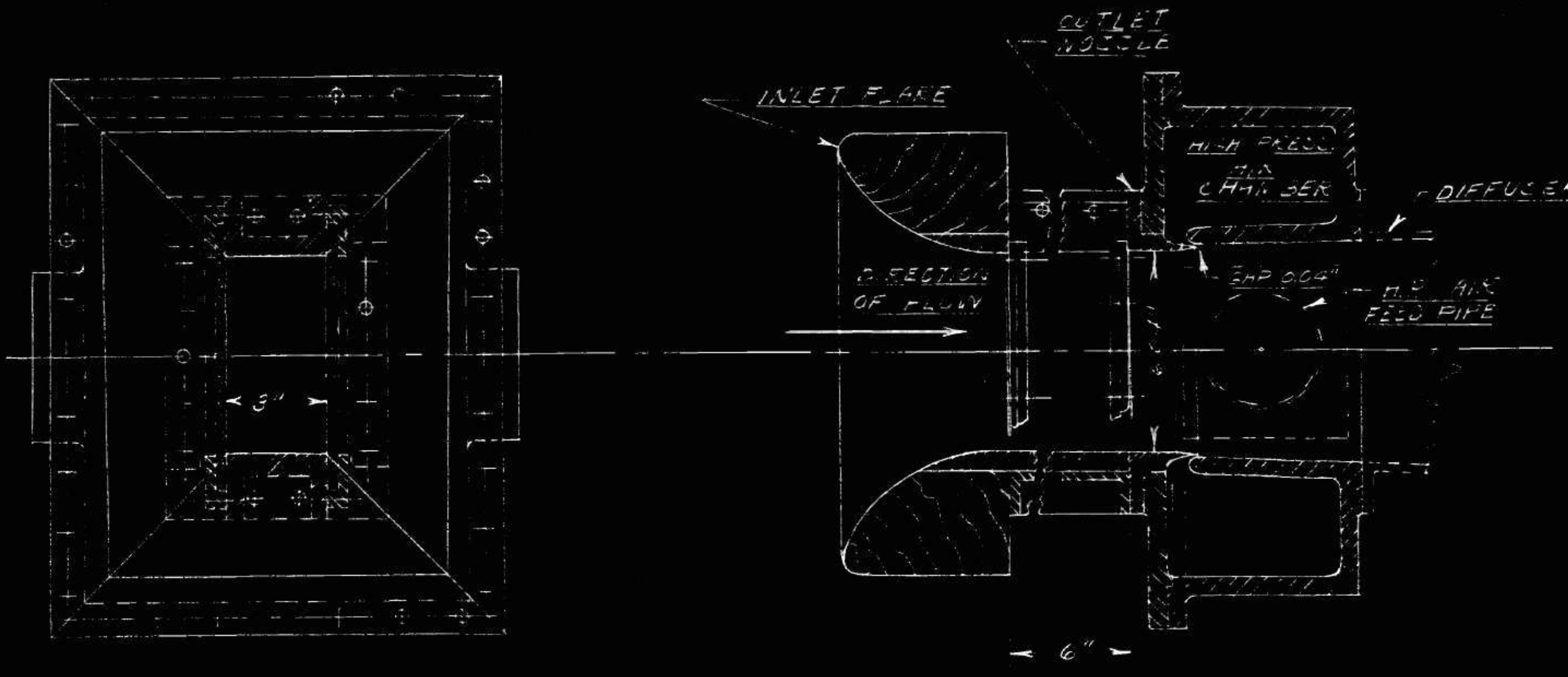
AUGMENTATION NECESSARY TO OBTAIN VARIOUS OVERALL EFFICIENCIES
EFFICIENCY OF COMPRESSOR NEGLECTED

$V = 900$ FT./SEC.

CIRCLED POINTS — VALUES OBTAINED USING VELOCITY, V, TO PRODUCE COMPRESSION

R.E.W.
4/20/42

FIG. 11



HIGH SPEED INDUCED FLOW WIND TUNNEL

[illegible]

FIG. 12

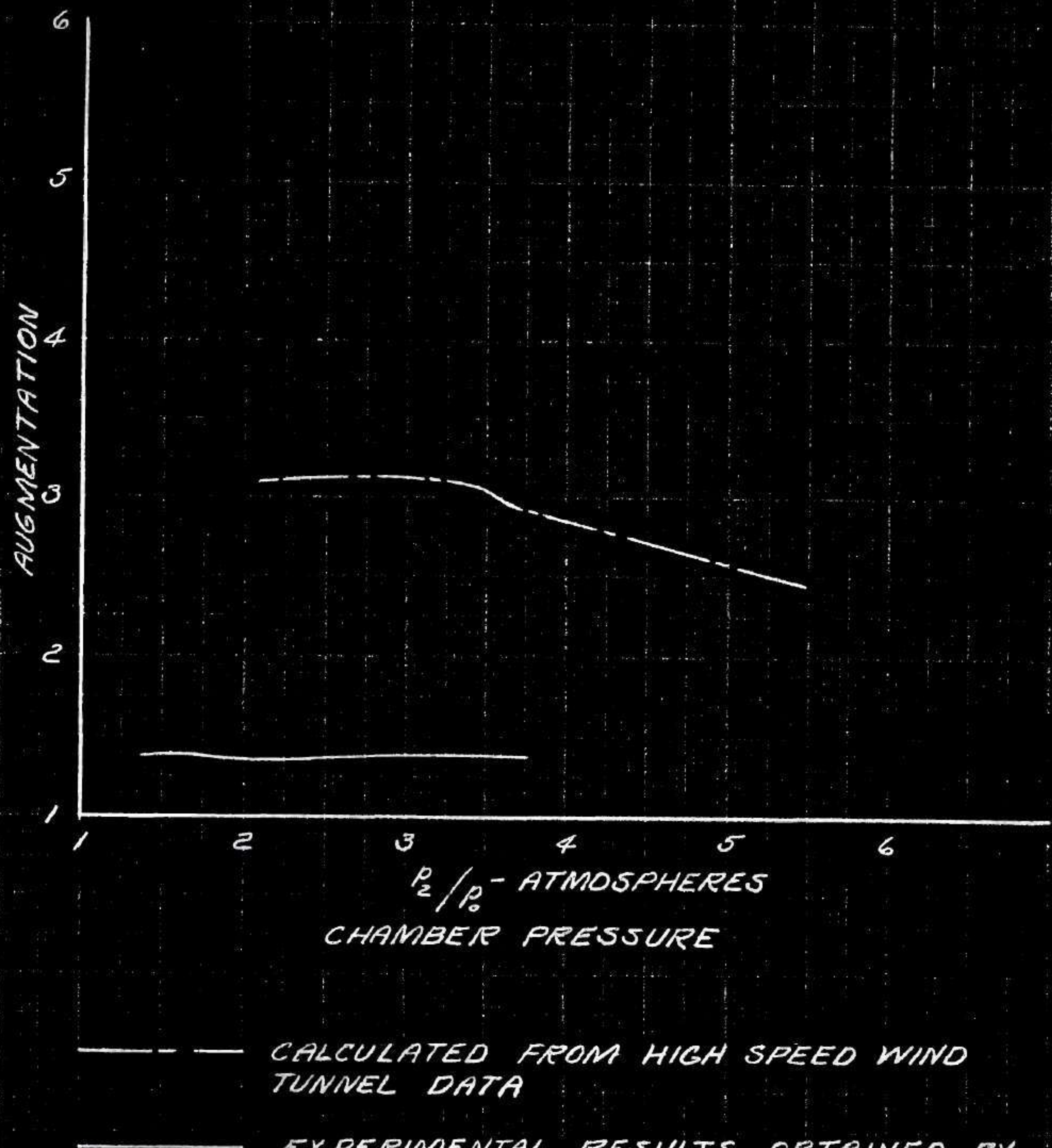
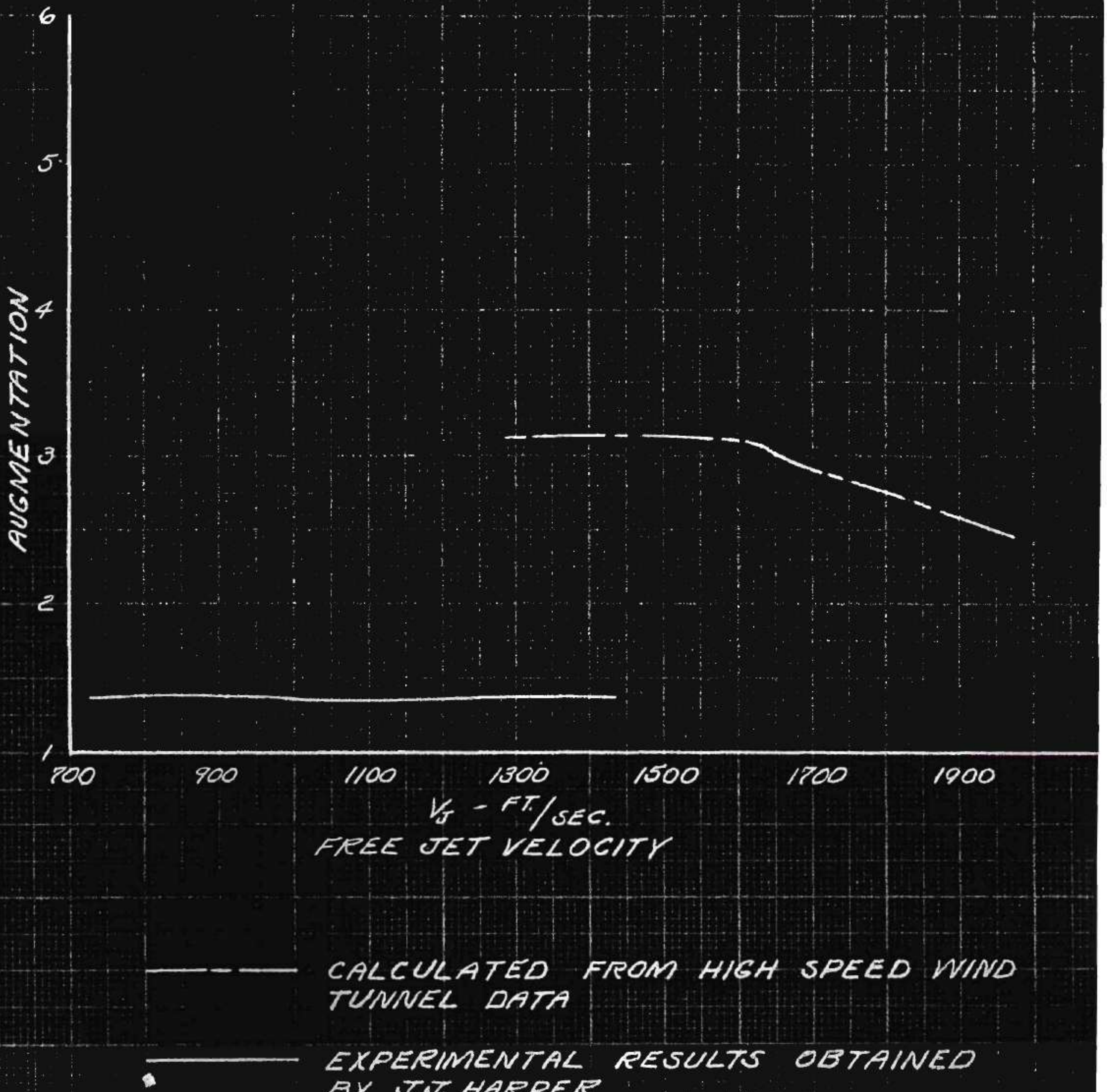

AUGMENTATION
6
5
4
3
2
1
1
2
3
4
5
6
P_2/P_0 - ATMOSPHERES
CHAMBER PRESSURE
CALCULATED FROM HIGH SPEED WIND TUNNEL DATA
EXPERIMENTAL RESULTS OBTAINED BY J.J. HARPER

FIG. 13


AUGMENTATION
1
2
3
4
5
6
700
900
1100
1300
1500
1700
1900
V_J - FT./SEC.
FREE JET VELOCITY
CALCULATED FROM HIGH SPEED WIND TUNNEL DATA
EXPERIMENTAL RESULTS OBTAINED BY J.J. HARPER